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WHITEHURST - INVARIABLE MEASURES OF LENGTH 1787

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J. H. Alexander.

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ATTEMPT

TOWARDS OBTAINING

INVARIABLE MEASURES.

1843. vij. 23

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A T T E M P T

TOWARDS OBTAINING

INVARIABLE MEASURES

OF

LENGTH, CAPACITY, AND WEIGHT,

FROM THE

MENSURATION OF TIME,

INDEPENDENT OF THE

MECHANICAL OPERATIONS

REQUISITE TO ASCERTAIN THE

CENTER OF OSCILLATION,

OR

THE TRUE LENGTH OF PENDULUMS.

By JOHN WHITEHURST, F. R. S.
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INTRODUCTION.

MANY diligent and elaborate refearches have been made by the learned to afcertain the origin, and proportions, of weights and meafures eftablifhed in various parts of the world. From thefe inquiries it appears, that feveral nations attempted to derive them from the weight and magnitude of natural bodies, or fundry parts of them, viz. the human ftature, cubit, fpan, leg, &c.; likewife from an hen's egg, grains of wheat, barley, and other fimilar productions. And thefe attempts were probably owing to an idea of the utility that might arife from the eftablifhment of fimilar *invariable* meafures of length, capacity, and weight, upon a natural foundation.

But the weight or magnitude of natural bodies of the fame denomination, being fubject to confider-

able inequalities, theſe ſeem to have fruſtrated the moſt ſagacious attempts to obtain invariable meaſures, and to have reduced the ſeveral ſtates and empires to the neceſſity of adopting arbitrary meaſures, according to the dictates of their own imaginations, without any foundation in the nature of things. It is therefore obvious that weights and meaſures, being thus derived and eſtabliſhed, can have no more relation or agreement with each other, than the languages of the ſeveral nations by whom they were adopted; and their being divided into a diverſity of ſmaller parts, renders them ſtill more incoherent and embarraſſing to thoſe nations who have any intercourſe in matters relative to ſcience, or commerce. See the tables annexed.

For although their relative proportions may be truly known, whereby the quantity of ſpace, magnitude, or weight, aſcertained by the meaſures of one nation, may be found correſpondent to a certain number of meaſures of another, yet it muſt be owned that ſuch operations not only demand an expence of time, but are alſo liable to error. It is therefore evident, that the diverſity of weights and meaſures eſtabliſhed by different nations, is no ſmall impediment to the progreſs of *ſcience and commerce.*

Weights and meaſures having been thus eſtabliſhed from time immemorial, have excited the attention of ſpeculative minds to obviate the difficulties and embarraſſments thence ariſing: for it appears to have been a principal deſideratum in philoſophical reſearches, to eſtabliſh weights and meaſures upon a permanent, unalterable foundation, whence invariable ſtandards might be obtained, to which all nations might refer, or with which they might compare their reſpective meaſures, and reduce them to one invariable, univerſal denomination, for the mutual convenience and benefit of all mankind: by deriving them from ſuch principles as might enable all future generations to obtain ſimilar meaſures of length, capacity, and weight, and thereby render it altogether needleſs to cut them on ſtone, or to engrave them in braſs, to perpetuate their exiſtence.

Such however, were apparently the motives which induced thoſe profound geometricians, Sir Chriſtopher Wren, and Mr. Huygens, to conſider the ſubject of invariable meaſures, and to ſuggeſt the idea of eſtabliſhing them upon the unerring laws of nature: namely, the periodical revolutions of the heavenly bodies, and the doctrine of pendulums. Conſidering

time as a natural meaſure, depending upon the equability of the earth's diurnal rotation, or the return of the ſame ſtar to the ſame meridian, in equal periods of time, at all times of the year, and in every year to come.

They likewiſe conſidered that the lengths of pendulums are inverſely as the ſquares of their reſpective vibrations in a given time; or that their vibrations in a given time are inverſely as the ſquare roots of their reſpective lengths.

Upon theſe conſtant, invariable laws of motion, thoſe celebrated philoſophers propoſed to have eſtabliſhed meaſures of length, viz. that the length of a pendulum, vibrating ſeconds of time, ſhould be taken for a ſtandard yard, meaſured from its point of ſuſpenſion to its center of oſcillation.

Such were the principles ſuggeſted by Sir Chriſtopher Wren and Mr. Huygens, as the baſis of invariable meaſures, obtainable in all future ages, *under ſimilar circumſtances of latitude, elevation, temperature*, &c.

But

But notwithſtanding that the principles above related are univerſally granted, yet, it muſt be owned that their application would have been attended with inſurmountable difficulties in the executive part; and with great inconveniency to the public; and theſe objections ſeem to have put a final period to the application of that excellent theoretical plan, for one century at leaſt. The impediments thence ariſing, are as follow:

Firſt, according to the plan laid down by mechanical writers for the purpoſe of aſcertaining the center of oſcillation, or the true length of pendulums, ſeveral menſurations are requiſite to be obtained with the greateſt degree of preciſion; and theſe are of ſo difficult a nature, that few, converſant in this buſineſs, would preſume to give ſimilar reſults from any two operations; but would rather previouſly conclude, that every attempt muſt be attended with an accumulation of error. The operations are to the following purport:

Let a ball that is perfectly *ſpherical, equally denſe*, and *of ſome of the heavieſt metals, be ſuſpended by a ſtring, the ſmalleſt, lighteſt, moſt flexible, and the*

leaſt

leaſt liable to ſtretch; let the pendulum, thus conſtructed, be adjuſted to vibrate ſeconds of time, or to perform 86400 vibrations in the ſpace of twenty-four hours mean time, or in one ſidereal day.

The pendulum being thus adjuſted, meaſure the diſtance between the point of ſuſpenſion and the center of the ball, and likewiſe the radius of the ball. Theſe proportions are given, to find a third proportional: thus — as the length, or diſtance between the point of ſuſpenſion and the center of the ball, is to the radius, ſo is the radius to the third proportional. The third proportional being thus found, and divided into five equal parts, two of them are required to be transferred from the center of the ball downwards, or, added to the length obtained by menſuration, give the diſtance between the point of ſuſpenſion, and the center of oſcillation, or the true length of the pendulum.

Such are the mechanical operations requiſite to aſcertain the length of pendulums, whatever may be the number of vibrations in a given time; therefore the difficulties attending the application of the above plan,

plan, are obvious to artifts practically converfant in dividing.

Secondly, the inconveniency arifing from the application of the above plan, is equally obvious when it is confidered, that the reputed length of the pendulum which vibrates feconds in the latitude of London, is 39,2 inches, and the ftandard yard is only 36 inches: for if the former was eftablifhed as a ftandard yard, it would certainly occafion a great alteration in the cuftomary meafures of England, and confequently create great confufion and inconveniency to the public.

This conjecture is abundantly verified by the quantity of time elapfed fince the promulgation thereof; for it does not appear that the fubject of invariable meafures was refumed again before the year 1771, when that learned philofopher, Mr. Andrew Bochem, fuggefted the idea of obtaining invariable meafures from the laws of motion, but in a very different manner; namely, by the admeafurement of the fpace heavy bodies defcend through in one fecond of time. See Act. Philof. Medic. Sec. Academ. Scient. Principal: Heffiæ. page 5, 1771: but this excellent theory, like the former, would have been attended with too much difficulty

difficulty in the executive part, to admit of a tolerable degree of precifion, when it is confidered that the velocity heavy bodies have acquired at the expiration of one fecond of time, is nearly 3,86 inches in $\frac{1}{100}$ part of a fecond; therefore the quantity of fpace bodies move through in one fecond, is not eafily afcertained by practice.

In the year 1774, the fociety inftituted for the encouragement of arts, manufactures, and commerce, took the fubject of invariable meafures into ferious confideration, at the inftance of Mr. Steel, a worthy member thereof, and advertifed the reward of a gold medal, or the fum of one hundred guineas to any perfon refiding in any country whatever, who fhould difcover and communicate to the fociety, on or before the third Tuefday in March, 1775, a mode whereby to obtain invariable ftandards for weights and meafures, communicable at all times, and to all nations; but the liberal encouragement thus held out to the public, was not productive of a fingle attempt; therefore the fame advertifement was repeated the following years, viz. 1776, 1777, and 1778. In confequence thereof, on the third Tuefday in March 1779, five plans were prefented to the fociety; amongft which,

which number, that invented by Mr. John Hatton, Watchmaker, in London, was the moſt approved, though not perfected to the degree of accuracy required in the conſtruction of invariable meaſures.

The ſuperiority of Mr. Hatton's plan conſiſted in the application of a moveable point of ſuſpenſion to one and the ſame pendulum, in order to produce the full and abſolute effect of two pendulums, the difference of whoſe lengths was the intended meaſure. But notwithſtanding the ſuperior excellency of Mr. Hatton's idea, it muſt be owned that the ingenious author was not equally happy in the mode of reducing it to practice. However, as the idea was new, and apparently capable of being carried to a much greater degree of perfection, the Society, in conſideration of its merit, and as ſome encouragement to re-conſider the ſubject, preſented him with the ſum of thirty guineas; and they alſo renewed their former advertiſement as a farther inducement to improve his plan, in hopes of obtaining the end propoſed the following year, but were diſappointed in their expectations; for ſeveral years elapſed, and no ſteps were apparently taken by Mr. Hatton towards a more effectual application of the principles he ſuggeſted; it was therefore

generally ſuppoſed the inventor of this machine had totally declined any farther conſideration of the ſubject. Theſe conſiderations, together with the favourable opinion I entertained of his ſcheme, induced me to attempt ſome improvement in the conſtuuction of Mr. Hatton's apparatus, in order to preſerve his idea from being too haſtily abandoned.

Having had the honour of attending at the exhibition of the ſeveral plans, at the requeſt of the Society, and of examining amongſt others the deſign and conſtruction of Mr. Hatton's apparatus, I became perſuaded in my own mind, that it was capable of being improved ſo far as to anſwer its purpoſe with a greater degree of accuracy than may be generally imagined; I was therefore induced to reſume the ſubject of invariable meaſures, and to attempt the conſtruction of an apparatus with which experiments have been making for ſeveral years paſt, which leave little room to doubt, but that meaſures of length may be obtained with preciſion; notwithſtanding the prejudice which prevails in the minds of many eminent philoſophers againſt the accuracy of pendulous experiments, and particularly againſt the application of maintaining powers, which ariſe from a conception that ſuch ap-

plications muſt neceſſarily accelerate, or retard, the vibrations, and render the reſult fallacious. And it muſt be owned, that according to the general conſtruction of clocks a century ago, they were liable to produce ſuch effects. But it is well known by thoſe who are converſant with the modern improvements of clockwork, that the maintaining power may be ſo applied as not to interfere in any ſenſible degree with the vibrations; and ſuch proviſion is made in the conſtruction of the apparatus as will effectually prove the truth, or fallacy, of ſuch conjectures.

For though ſundry impediments unavoidably accompany the conſtruction of time-keepers, and interfere in ſome degree with the menſuration of time, ſo as to exclude the reſult of ſuch experiments from any pretenſions to mathematical exactneſs; yet it evidently appears, from a ſeries of ſeveral years experiments, that meaſures may be obtained from the menſuration of time, ſufficiently accurate for the various purpoſes of human life. To what degree of accuracy ſuch machines are capable of being brought, will beſt appear from the experiments themſelves.

During the courfe of the obfervations, we have been under the neceffity of proceeding without the aid of a tranfit inftrument, though fo effentially neceffary for the adjuftment of the pendulums; on which account no journal has been kept of the time, temperature, &c. which might have been naturally expected. But as a fubftitute for that inftrument, Mr. Dutton has, in a very obliging manner, indulged me with free accefs to his regulator, upon which the menfuration of time, by the two pendulums, has been adjufted; therefore I do not apprehend that any confiderable error has arifen, or indeed could arife from that circumftance, in the meafure obtained.

From the principles here adopted, the length of the feconds pendulum, or the force of gravity, in different latitudes, may be obtained with great precifion.

Upon the whole, I would fain hope that this little attempt will not be deemed unworthy the patronage and protection of all real lovers of fcience and commerce.

As the machine which Mr. Hatton invented is carefully preferved in the Society's mufeum, for the infpection of the public, it is needlefs to defcribe its conftruction, or to point out the inftances wherein the apparatus does not effectually anfwer the end propofed.

For the fake of brevity, the following account contains no more than a defcription of the apparatus in its improved ftate, and not a detail of the various fteps which have been taken to obtain its prefent degree of perfection.

AN ATTEMPT

AN

ATTEMPT

TOWARDS OBTAINING

INVARIABLE MEASURES

OF

LENGTH, CAPACITY, AND WEIGHT,

FROM THE

MENSURATION OF TIME,

Independent of the mechanical Operations requisite to ascertain the Center of Oscillation, or the true Length of

PENDULUMS.

SECTION I.

Of the general Design of the Apparatus.

PREVIOUS to a description of the apparatus adapted to the purpose of obtaining invariable measures, it may not be improper to premise the principal objects of its construction:

First,

Firſt, it is propoſed to obtain a meaſure of the greateſt length conveniency will permit, from the menſuration of time, with eaſe and certainty; preſuming that a meaſure of conſiderable length may be ſubdivided with a much greater degree of accuracy, than a ſhort one can be multiplied into a longer.

Secondly, it is propoſed to obtain the meaſure from two pendulums, whoſe vibrations are to each other as two to one, and whoſe lengths coincide with the Engliſh ſtandard in *whole numbers*, either feet or inches; in order to ſimplify the operations depending upon them, and alſo with a view of eſtabliſhing the Britiſh meaſures upon a *natural permanent foundation*, which may enable all future generations to obtain ſimilar meaſures with eaſe and certainty.

Such are the objects to be obtained by the conſtruction of the apparatus.

According to the doctrine of pendulums, the number of vibrations performed in a given time, are inverſely as the ſquare roots of their reſpective lengths; and yet we find but few inſtances wherein the lengths of pendulums coinciding with Engliſh meaſure,

ſure, whoſe vibrations in a given time, conſiſt of whole, and diviſible numbers ; and yet ſuch lengths and vibrations only, will anſwer the purpoſe required.

Then ſince the aſſumed length of the ſeconds pendulum in the latitude of London, is 39,2 inches; and according to the reſult of the following theorem, pendulums which vibrate 42 and 84 times in a minute coincide with Engliſh meaſure, viz:

$$\left.\begin{array}{l} 42^2 : 60^2 :: 39,2 : 80 \\ 84^2 : 60^2 :: 39,2 : 20 \end{array}\right\}\text{Inches.}$$

Such are the lengths and vibrations of the two pendulums applied in the conſtruction of the apparatus, and the difference of their lengths is the meaſure propoſed to be obtained, viz. 80—20=60 inches; and this, independent of the mechanical operations requiſite to aſcertain the center of oſcillation, or the true length of pendulums.

The reſult of this reaſoning would have been ſtrictly true, provided the ball were infinitely ſmall, and the aſſumed length of the ſecond's pendulum were alſo true, viz =39,2 inches. But ſince the accu-

racy of theſe experiments depends very much on the momentum of the ball overcoming, in ſome degree, the reſiſtance of the medium, and alſo the inequalities of the impetus on the pendulum, it becomes requiſite to apply ſuch a quantity of matter in the magnitude thereof, as will effectually anſwer thoſe neceſſary purpoſes. Conſidering likewiſe, that great advantages may hereafter ariſe from the magnitude of the pendulum ball correſponding with an aliquot part of the length of each pendulum; and that a metallick globe, two inches diameter, not only coincides with an aliquot part of the length, but may alſo be conſidered as containing a ſufficient quantity of matter to produce the momentum required for the accuracy of the experiments. Hence a globe according to the above magnitude is particularly choſen for the pendulum ball; for its radius being one inch, or rather $\frac{1}{80}$ of the longer pendulum, and $\frac{1}{20}$ of the ſhorter, this renders it more capable of being obtained with facility in any future age by approximation, from the menſuration of time alone, than if it conſiſted of fractional parts of the length thereof.

Such were the motives which induced me to apply a pendulum ball, radius $=$ 1 inch.

Now

Now the radius of the ball being 1 inch, and the diſtance between the centers of oſcillation of the two pendulums and the center of the ball, being in the inverſe ratio of their reſpective lengths, renders the above computation not ſtrictly correct — Theſe are truths well known to mathematicians: but more of this in its due place.

It is likewiſe equally well known, that the times of vibration are only iſochronal whilſt the pendulum deſcribes ſimilar arcs. Since their times are directly as 8 R + V, or as 8 times the radius added to the verſed ſign of the arc deſcribed; or as the number 51840 added to the ſquare of the number of degrees contained in the arc: it is therefore evident that pendulums actuated by the force of gravity alone, independent of a maintaining power, whoſe arcs of vibration gradually diminiſh till they come to reſt, cannot be adjuſted with the ſame degree of accuracy as thoſe which deſcribe equal arcs for the ſpace of ſeveral weeks, months, or years.

Whence it appears that the application of a maintaining power becomes abſolutely neceſſary, to com-

penſate for the reſiſtance of the medium, and which may neither accelerate nor retard the vibrations; but the poſſibility of ſuch an application as will effectually anſwer thoſe important purpoſes, has been much doubted; therefore, as ſuch opinions, however erroneous, may poſſibly prejudice the minds of many againſt the accuracy of ſuch experiments, and as the propoſition aſſerted is rather to be demonſtrated by experiment than by reaſon; proviſion is made in the conſtruction of the apparatus, whereby the truth or fallacy of ſuch conjectures may be truly aſcertained.

And in order to ſhew whether the plan adopted i capable of being reduced to general practice, the movement or maintaining power is not made eſſentially different from that of a common eight-day clock, nor is it executed with more than common care; preſuming that if the reſult of the experiments depended upon too much accuracy in the executive part of the apparatus, the plan would be the leſs generally uſeful.

Therefore the train conſiſts of the uſual number of wheels and pinions, containing the uſual number of teeth, the firſt pinion excepted, being No. 12, as a means of rendering the impetus on the pendulum ra-

ther more equable, and the pendulum wheel No. 21, in conformity to the vibrations of the pendulums.

The dead ſcapement is applied, with the addition of a counterpoiſe to the pallats, in order to adjuſt their center of gravity, and render their vibrations coincident with thoſe of each pendulum, viz. 42 and 84 in a minute, and this in order to remove their ſuppoſed tendency to accelerate or retard the vibrations of the pendulum.

The pendulum is thus conſtructed: it conſiſts of a *ſolid, ſpherical, leaden ball*, two inches diameter, its weight 25 oz. 10 dwt. 11 gr. or 12251 grains troy weight, ſuſpended by *a flat ſteel wire, tempered*, 80 inches of which is nearly equal to 3 grains.

Such is the conſtruction of the pendulum; and it has already been obſerved that the magnitude of the ball is particularly choſen as being an aliquot part of the length of each pendulum, and more eaſily obtainable by approximation from the menſuration of time, and therefore it becomes more immediately inſtrumental in perpetuating the exiſtence of the meaſures thus derived; for although a little variation in the magnitude

tude of the ball will only produce an infenfible error in the length of the ftandard meafure, yet it neverthelefs becomes defirable to proceed by fimilar means to obtain fimilar refults; therefore it is an object of fome importance to apply the above radius, viz. $\frac{1}{80}$ of the longer pendulum, and $\frac{1}{20}$ of the fhorter, as the only means of perpetuating the exiftance of the fame meafure to the end of time.

It has been objected by fome that the quantity of cohefion contained in fo fmall a quantity of matter as that of three grains in a wire = 80 inches, is not capable of fuftaining the weight of the ball without ftretching, but if an equal menfuration of time, for the fpace of feveral months or years, is any proof to the contrary, fuch teftimony is not wanting.

Having premifed the general plan of the apparatus, it now remains to give a particular defcription of its conftruction.

SEC-

SECTION II.

Of the Conſtruction of the Apparatus.

LET *a a a a* b b plate I, fig. 1, repreſent a wooden frame, made of two-inch deal plank, ſtrait grained, according to the ſcale annexed. Deal is particularly choſen in preference to many other ſubſtances as being leſs liable to expand and contract length way of the grain than the generality of timber.

The ſtrength of the frame is intended to give greater ſtability to the vibrations, and render the reſult of the experiments more certain. p h repreſents a braſs ruler about five feet two inches long, and $\frac{1}{4}$ of an inch thick, let into the plank, till its upper ſurface coincides with the plane thereof. Its lower end is permanently fixed by a ſcrew at p, its upper end is alſo fixed by a ſcrew at h, but with full liberty of expand-

expanding or contracting according to the temperature of the feafons.

Fig. 2, reprefents the fame frame with a part of the apparatus annexed. A a ftrong brafs plate, to which the movement or maintaining power is connected by means of four pillars or dial feet. To the fame plate the moveable point of fufpenfion m is permanently fixed, between the chops of which the pendulum wire paffes, and in which it is fixed at pleafure by means of a fcrew compreffing them together. To the fame plate is alfo fixed a fcale of degrees to meafure the arcs of vibration. T the fituation of the cord barrel ; H the pulley over which the line paffes to fufpend the weight or power W ; p h the brafs rule before defcribed. But the parts are fo much reduced to fhew their feveral relations or fituations upon the frame, that they are thereby rendered too minute to be well defined ; therefore they are reprefented according to their real magnitudes in the following plate.

Plate II, fig. 2, reprefents the plate A before defcribed; N the moveable point of fufpenfion, c c its chops, between the parts of which the pendulum wire paffes

paſſes, and in which it is permanently fixed at pleaſure by means of the ſcrew S, as it were in the chops of a vice. D D D D four pillars or dial feet $\frac{3}{4}$ of an inch long, whereby the movement or maintaining power is connected with, or detached from the pendulum at pleaſure. M M the ſcale of degrees, to meaſure the arcs of vibrations. R R R the braſs ruler before deſcribed, lying under the plate A, and alſo under the apparatus L, plate II, fig. 1. By means of this apparatus the long pendulum is adjuſted. B a braſs cylinder through which a ſteel ſcrew *a a* paſſes, the end of which reſts upon L, fig. 1, at y. o o a part of the cylinder which projects at right-angles from it, and upon which projection the pendulum is ſuſpended when the wire is at liberty in c c. Now o o being raiſed or lowered by the ſteel ſcrew, lengthens or ſhortens the longer pendulum, by previouſly opening the chops of the moveable point of ſuſpenſion: the pendulum wire being fixed in q q by means of a pin.

At the upper end of the ſcrew *a a*, a wheel No. 25, is fixed for the nicer adjuſtment of the pendulum, one tooth being $= \frac{1}{1000}$ of an inch in the length of the pendulum.

The plate A, Fig. 2, is fixed to the frame as before directed. The two pillars being inserted at P P for that purpose; and the apparatus L is immovably fixed to the frame by means of several screws.

L, Fig. 5, represents a transverse section of the plate A, to which the moveable point of suspension N is connected, and in which the pendulum wire is inclosed; whence some idea may be formed of its stability. *d d*, fig. 4, represents the crutch through which the wire passes; *c c* the counterpoise to the pallats; *b* the moveable ball whereby their center of gravity may be adjusted, to render the vibration equal to those of each pendulum, viz. 42 and 84 per minute.

Fig. 3 represents the form of the pillars which are inserted into the plate A, and pass between the planks to connect the apparatus to the frame.

Fig. 6 represents the instrument adapted to draw the two lines on the ruler when the pendulums are adjusted: but more of this hereafter.

Plate III represents not only the dial plate, but also the front plate of the movement frame; the indexes

dexes being fixed on the pivots of the firft, and pendulum wheel pinions. *c c* the counterpoife; *b* the moveable ball, applied to adjuft the vibrations of the pallats.

The preceding account of the apparatus may ferve to convey fome idea of its conftruction, to thofe who are converfant with the mechanifm of clocks or watches; therefore it may be altogether needlefs to attempt a more explicit defcription thereof; however it may be neceffary to give fome account of the mode by which it has been applied.

SECTION III.

Concerning the Application of the Apparatus.

1. IT may not be improper to obſerve, that it was found convenient to fix the apparatus in a diagonal direction in the corner of a room, ſo far detached from the walls as to allow ſufficient room for one's hand to paſs behind the frame, to fix or detach the apparatus as occaſion may require.

2. The frame being connected to the walls at the upper and lower ends, renders it ſufficiently permanent. The braſs ruler R R R being fixed as before directed, and alſo the adjuſting apparatus L, fig. 1, plate II. together with the plate A, fig. 2, let the pendulum be ſuſpended upon o o, fig. 1, its wire paſſing through the chops of the moveable point of ſuſpenſion c c.

3. As preparative to the menſuration of time, reſt the pallats upon a proper fulcrum, and adjuſt their

vibrations to coincide with thofe of the pendulums, viz. 42 and 84 times in a minute, by means of the ball *b*, and make fuch marks on the arm of the lever *c c*, as the ball may be returned to the fame place hereafter.

4. The vibrations of the pallats being thus adjufted, fet the ball *b* to perform 42 vibrations in a minute, and having applied the maintaining power to the pendulum, proceed to experiment.

5. The pendulum being then adjufted to perform 42 vibrations per minute, or 60480 times in the fpace of 24 hours mean time, and this under a temperature 60° on Fahrenheit's fcale.

6. The apparatus being thus prepared, let a line be drawn upon the brafs ruler R R R, along the edge of the plate A, fig. 2, plate II, at Y, with the inftrument A, fig. 6, the open end S paffing underneath the head of the fcrew S, plate III, and the other end bearing upon the plate A, at Y, plate II. By this means it is prefumed that no error can arife by drawing the lines, as might have been the cafe, from the hand alone.

7. Note

7. Note the quantity of the arc defcribed by the pendnlum, viz. 3° 20′, difengage the wire in c c, and detach the maintaining power from the frame, and flide it down to reft upon the point of the fcrew o, plate I, fig. 1. The apparatus being refixed to the frame, and alfo the wire in the moveable point of fufpenfion, proceed to the fecond adjuftment of the pendulum, by means of the fcrew o, plate I, fig. 1, having previoufly removed the ball *b*, to vibrate 84 times per minute.

8. The movement being thus prepared for a fecond experiment, let the pendulum be adjufted to perform 84 vibrations per minute, or 120960 in the fpace of 24 hours mean time, and draw another line upon the ruler as before directed, and note the arc of vibration, viz. 3°: 20′; it being requifite that the arcs of vibration fhould be fimilar to the former, and performed in the fame temperature of air.

Thus meafures of length, nearly equal to five feet Englifh meafure, may be obtained from the menfuration of time, with much more accuracy than has generally been imagined; and this independent of the me-

mechanical operations requisite to ascertain the center of oscillation, or the true length of pendulums.

The measure or space between the two marks thus obtained, is not 60 inches, according to the result of the preceding problem, but 59,892 inches, according to Mr. Troughton's standard, which seems to indicate that the pendulums were not accelerated by the maintaining power, as is generally supposed; since the lengths of the pendulums must have been increased to compensate for that effect in the mensuration of time. However that may be, such is the result of the experiments; whence it appears that the length of the pendulum vibrating seconds of time in the latitude of Landon, has been computed and found = 39,1196 inches, and not 39,2 inches, as commonly supposed.

see Ph. Tr. lxxxviii. 135. where Sir G. Shuckburgh, having re-measured these marks found the space = 59.89358 in. tem. 64° by Troughton's scale made after Bird's standard.

Kater's determin. with a convertible pend. = 39.13929 in. Ph. Tr. 1818.

According to the above datum, or length of the second's pendulum, viz. 39,1196 inches, it appears that heavy bodies descend through the space of 16,087 feet in one second of time, which is near 16 feet and one inch; therefore since the result of the preceding experiments so nearly coincide with the laws of motion, may we not thence infer that the maintaining

ing power has no improper effect on the vibrations of the pendulums; but only compenſates for the reſiſtance of the medium, and neither accelerates nor retards the vibrations? However, ſuch are the effects produced, and therefore I have not the leaſt reaſon to doubt but that future experiments will abundantly confirm the truth of the former, and fully perſuade mankind that invariable meaſures may be obtained from the menſuration of time, under ſimilar circumſtances of latitude, elevation, temperature, &c.; and that by the ſame means the force of gravity, or the true length of a pendulum vibrating ſeconds of time, may be aſcertained with a conſiderable degree of preciſion in all latitudes.

Having previouſly obſerved that the two pendulums deſcribed ſimilar arcs, viv. $3^{\circ} : 20'$ nearly, it may not be improper to take notice, that in order to produce thoſe effects, the apparatus requires powers proportional to the reſpective lengths of each pendulum, viz. the 80-inch pendulum a power = 32 ounces troy weight; and that of 20 inches, a power = 8 ounces.

SECTION IV.

Of Meaſures of Capacity and Meaſures of Weight.

HAVING premiſed the conſtruction and the application of the apparatus, and compared the reſult of the preceding experiments with the Engliſh ſtandard, we have now to conſider the moſt effectual mode of obtaining meaſures of capacity and meaſures of weight, from the ſpace obtained from the menſuration of time, and this with a view of rendering their ſeveral relations ſubſervient to each other, at any future period of time, if occaſion ſhould require.

It has already been obſerved that the meaſure or ſpace obtained from the menſuration of time is 59,892 inches Engliſh meaſure; which we purpoſe dividing into five equal parts, and each of thoſe parts into ten.

And as the meaſure thus graduated is derived from the laws of nature, it may not be improperly diſtinguiſhed by the appellation of *a philoſophical meaſure*, in contradiſtinction to all other meaſures now exiſting.

The diviſion of the foot into decimal parts, is not altogether adopted to facilitate the buſineſs of arithmetical computation, but rather in conſequence of the number 10 being the cube root of 1000. For ſince a cubic foot of rain or diſtilled water is 1000 ounces, avoirdupois weight, 1 cubic inch of the above meaſure will = 1 ounce.

Hence it is propoſed to obtain original ſtandards for weights nearly correſponding with the Engliſh avoirdupois weight; and, as ſuch, may be conſidered as a permanent baſis to found the latter upon: by which means the relation between thoſe of length, capacity, and weight, may be effectually preſerved; inſomuch, that the former might be derived from the latter, if occaſion ſo required.

In order to obtain meaſures of weight from thoſe of length, let us ſuppoſe a cubic veſſel accurately

rately conſtructed from the above meaſure, to conſiſt of any number of cubic inches, which are diviſible to unity, and let the weight of the water contained therein be taken for an original ſtandard weight.

Now, ſince the weight of one cubic foot of *rain*, or *diſtilled* water, is 1000 ounces avoirdupois weight, one cubic inch of philoſophical meaſure becomes equal to one ounce: therefore if a cubic veſſel is conſtructed to contain 64, or 512 cubic inches, the contents of the former becomes = 64, and the latter = 512 ounces: theſe numbers being diviſible to unity, either of them may be taken for an original ſtandard weight, capable of being ſubdivided by means of a good balance, or multiplied to any greater extent: but let it be remembered that the accuracy of theſe experiments, like the former, depend on the ſame temperature, viz. 60°.

Such are the modes propoſed for obtaining meaſures of length, capacity, and weight, from the menſuration of time: therefore, ſince all the operations depend on the application of whole numbers, they are thereby rendered more ſimple, and leſs liable to

error than if they depended in any degree on fractional parts. And being thus derived, their analogy, or relation to each other, becomes the more obvious.

Now, ſince it appears that avoirdupois weight may be thus eſtabliſhed upon a permanent, unalterable foundation, may it not with propriety be adopted as an invariable ſtandard for troy weight? For, having once truly compared the weights of theſe different denominations together, and aſcertained their relative proportions, viz. the number of grains troy contained in ten, twenty, or any other number of ounces avoirdupois, the latter becomes thereby an invariable ſtandard for the former.

Having premiſed theſe matters we have now to conſider the conſtruction of a cubic veſſel, as an original ſtandard for weights.

It has already been obſerved, that the contents of a veſſel = 64, or 512 cubic inches, may be conveniently adopted for that purpoſe, as being diviſible to unity; but whether the former or the latter will admit of the greateſt degree of accuracy in the executive

ecutive part, I leave to the decifion of the artift who may undertake the conftruction of it. However it may not be improper to add, that fince the fuperficial contents of the larger veffel is to that of the fmaller as 4 to 1, while their folid contents are as 8 to 1; the former may poffibly admit of more accuracy in the executive part than the latter.

In order to avoid any error that might arife from the inaccuracy of filling an open veffel with water, it is propofed to conftruct a cubic veffel confifting of fix complete fides, and to perforate one of its corners for the admiffion of a funnel, and alfo to fix the veffel in a diagonal direction to the horizon. In this pofition the veffel will receive its water, fo as to expel the contained air. The weight of the veffel having been previoufly afcertained, the additional weight arifing from the water contained therein, when completely full, becomes an original, invariable ftandard for avoirdupois weight; obtainable in all future ages, under fimilar circumftances of temperature, &c., to the end of time: for it is reafonable to fuppofe, that, from fimilar caufes, fimilar effects will arife.

But, however that may be in reality, fuch are the ideas

ideas which have hitherto occured on that ſubject relative to the original conſtruction of meaſures, both of length, capacity, and weight: for ſince it appears, from ſundry experiments, that an alteration $= \frac{1}{1000}$ of an inch in the pendulum whoſe length is 80 inches, will produce a ſenſible alteration in its menſuration of time; may we not thence infer, that the reſult of ſuch experiments, with the aid of a tranſit-inſtrument, muſt be attended with a conſiderable degree of accuracy? for ſince the meaſures of capacity and weight deduced from thence, depend altogether upon whole numbers, it is reaſonable to expect that a conſiderable analogy in the accuracy of their conſtruction may enſue.

The moſt uſeful and moſt advantageous application of weights and meaſures being a ſubject univerſally known, with regard to their ſeveral diviſions and combinations, it ſeems altogether needleſs to ſuggeſt a ſingle idea on that head. I ſhall therefore proceed to give ſome account of ſundry experiments which have been made to aſcertain the effects produced by the maintaining power on the vibrations of the pendulum.

SEC-

SECTION V.

Of ſundry Experiments tending to aſcertain the Effects produced on the Vibration of the Pendulum by the maintaining Power.

PROVISION having been made in the conſtruction of the apparatus, with a view of aſcertaining the influence of the maintaining power on the vibrations of the pendulum, the following experiments have been repeatedly made with ſimilar reſults:

I. It may be neceſſary to obſerve that the apparatus is purpoſely conſtructed to detach the maintaining power from the pendulum without interfering in any degree with the vibrations.

II. When the pendulum has been perfectly adjuſted to perform 42 vibrations per minute, and has been going at that rate for the ſpace of ſeveral days or

weeks, and at the ſame time deſcribing an arc of 3°: 20′, nearly.

Under thoſe circumſtances the maintaining power has been detached from the pendulum, and its vibrations have been carefully obſerved and compared by thoſe of a pendulum vibrating ſeconds, by which the former was adjuſted — And the following effects were produced, viz.; in the ſpace of 62 minutes the arc of vibration diminiſhed from 3°: 20′, to 1°: 40′; and the time gained from the diminution of the arcs, nearly half one vibration.

Such having been the reſult of ſeveral experiments, render the effects produced on the pendulum ſubject to computation, according to the rules laid down page 5: whence it appears, that the maintaining power only compenſates for the reſiſtance of the medium, and neither accelerates nor retards the vibrations, though a contrary opinion has generally prevailed.

And as ſome farther teſtimony of the efficacy accompanying the urging or maintaining power, an ingenious Mathematician* has favoured me with the

* John Rotheram, M. D.

follow-

following problem computed from the defcription of the apparatus, and the refult of feveral experiments made therewith. The problem having been examined and approved by Charles Hutton, LL. D. profeffor of mathematics in the Royal Academy of Woolwich, I prefume it may be acceptable to many mathematical readers.

P R O B L E M.

" THE meafure is the difference of the lengths " of two pendulums whofe times of vibration are " known to be 84 and 42 in a minute: from this " datum the accurate length of a feconds pendulum " may be obtained. For let $y =$ the length of the " fhorter pendulum, viz. the diftance between the " point of fufpenfion and the center of the ball. " Then $y + d =$ the diftance between the point of " fufpenfion and the center of the ball of the longer " pendulum; d being $=$ the diftance between the " two points of fufpenfion $= 59{,}892$ inches.

" Now

" Now the radius of the ball being one inch, and " the weight of the rod, or the fuſpenſion wire, be- " ing almoſt infinitely ſmall in proportion to the " weight of the ball, by the deſcription of the appa- " ratus; the diſtance between the center of the ball " of the ſhorter pendulum and its center of oſcilla- " tion will be expreſſed by $\frac{2}{5y}$. Vid. Emmerſon's " Fluxions, § 2. prob. 18. exampl. 16.; ſo that the " true length of the ſhorter pendulum, viz. the diſ- " tance of the point of ſuſpenſion from the center " of oſcillation, will be $y + \frac{2}{5y}$. And by the ſame " reaſoning the true length of the longer pendulum " will be $y + d + \frac{2}{5y + 5d}$. But by mechanics, the " lengths of pendulums are as the ſquares of the " times of vibrations, and in this caſe the time of " one vibration is twice that of the other: therefore " $y + \frac{2}{5y} : y + d + \frac{2}{5y + 5d} :: 1^2 : 2^2$; from whence " this equation, $4y + \frac{8}{5y} = y + d + \frac{2}{5y + 5d}$, which " gives, after proper reductions, $y = 19{,}939$ inches " the length of a pendulum to beat 84 times in a " minute; then to find the length of a pendulum to " beat ſeconds,

" $60^2 : 84^2 :: 19{,}959 : 39{,}1196$ inches the " length of the ſeconds pendulum.

" From

" From whence we obtain the ſpace heavy bodies " deſcend through in a ſecond : for by mechanics, " the time of one vibration : the time of a body's " fall through $\frac{1}{2}$ the length of the pendulum : : " the circumference of a circle : the diameter, viz. " : : 3,1415, &c. : 1 ; and the ſpaces are as the " ſquares of the times : therefore $1^2 : \overline{3{,}1415\ \&c.}^2 :: \frac{39{,}1196}{2} : 193{,}047$ inches $= 16{,}087$ feet, or 16 " feet and an inch very nearly."

Therefore ſince the reſult of this reaſoning ſo nearly coincides with the laws of motion, with reſpect to the ſpace heavy bodies fall through in one ſecond of time, may we not thence infer, that a conſiderable degree of accuracy may be obtained from the plan adopted? inſomuch as to preſume that future experiments may eſtabliſh this buſineſs upon a permanent foundation—But however that may be, I have the ſatisfaction to lay before the public a faithful account of the progreſs I have made in this neceſſary work, though conſcious much more remains to be done than either my abilities or leiſure will permit.

The following tables are only inſerted to ſhew, in ſome degree, the preſent ſtate of weights and mea-

ſures, eſtabliſhed in many parts of Europe; whence ſome idea may be formed relative to the great embarraſments daily ariſing from the diverſity and incoherence of them in matters of ſcience and commerce.

P. S. Omited in page 23, to mention, that original ſtandard weights may alſo be obtained hydroſtatically, by immerging a metalic cube, conſtructed according to the preceding dimenſions, in water, temperature = 60°.

A Table

A TABLE *of foreign Meaſures carefully compared with the Engliſh; ſhewing the great diverſity of their Lengths, and the Number of Parts into which each Foot, Ell, &c., is divided.*

Suppoſe the Engliſh foot divided into 1000 equal parts, thoſe here mentioned are in proportion to it, as follows:

Places.	Foot compared.	Length of Feet in Engliſh Inches and Tenths.
London - - - - -	1000	12,0
Paris, Royal - - -	1068	2,8
Amſterdam - - - -	942	11,3
Brill - - - - - -	1103	13,2
Antwerp - - - -	946	11,3
Dort - - - - - -	1184	14,2
Rynland or Leyden -	1033	12,4
Lorain - - - - -	958	11,4
Mecklin - - - -	919	11,0
Middleburg - - - -	991	11,9
Straſbourg - - - -	920	11,0
Bremin - - - - -	964	11,6
Cologn - - - - -	954	11,4
Frankfort ad Manum -	948	11,4
Spaniſh - - - - -	1001	12,0
Toledo - - - - -	899	10,7
Roman - - - - -	967	11,6

Bononia

Places.	Proportion of Foot.	Length of Feet in English Inches and Tenths.
Bononia - - - - -	1204	12, 4
Mantua - - - - -	1569	18, 8
Venice - - - - -	1162	13, 9
Dantzick - - - -	944	11, 3
Copenhagen - - -	965	11, 6
Prague - - - - -	1026	12, 3
Riga - - - - - -	1831	21, 9
Turin - - - - - -	1062	12, 7
The Greek - - - -	1007	12, 1
Old Roman - - -	970	11, 6
Bononian - - - -	1140	13, 7

Places.	Ell.	Ft. Inc.
Lyons - - - - - -	3976	3 11, 7
Boulogne - - - - -	2056	2 0, 8
Amſterdam - - - -	2269	2 3, 2
Antwerp - - - -	2273	2 0, 2
Rynland or Leyden - -	2260	2 3, 1
Frankfort - - - -	1826	1 9, 9
Hamburgh - - -	1905	1 10, 8
Leipſick - - - - -	2260	2 3, 1
Lubeck - - - -	1908	1 9, 8
Nuremburg - - -	2227	2 3, 3
Bavaria - - - - -	954	0 11, 4
Vienna - - - - -	1053	1 0, 6
Bononia - - - - -	2147	2 1, 7
Dunkirk - - - -	1903	1 0, 8
Florence - - - -	1913	1 11, 0

A TABLE

A TABLE *of the foreign Weights compared with the English Pound Avoirdupois, divided into* 100 *equal Parts.*

Places.	Pound.
London - - - -	100
Paris - - - - -	93
Lyons - - - - -	109
Boulogne - - - - -	89
Amſterdam - - - -	93
Antwerp - - - -	98
Leyden - - - - -	96
Lorrain - - - - -	98
Mecklin - - - -	98
Middleburg - - -	98
Straſbourg - - -	93
Bremen - - - - -	94
Cologn - - - - -	97
Frankfort - - - -	93
Hamburgh - - -	95
Leipſick - - - -	115
Nuremburg - - -	94
Vienna - - - - -	83
Caſtile - - - - -	99
Liſbon - - - - -	106
Gibraltar - - - -	103
Toledo - - - - -	100
Rome - - - - -	123
Bononia - - - -	127

Florence

Places.	Pound.
Florence - - - -	123
Naples - - - - -	143
Genoa - - - - -	142
Mantua - - - - -	143
Milan - - - - -	140
Parma - - - - -	143
Venice - - - - -	153
Dantzick - - - -	119
Copenhagen - - -	94
Prague - - - - -	106
Cairo - - - - -	161
Conſtantionople - -	86
See Harris's Lexicon Technicum.	

The inſtances above recited, evidently ſhew that ſimilar meaſures of length, capacity, and weight, muſt neceſſarily contribute to the mutual benefit of all nations.

THE END.

ERRATA.

Page 3, Line 9, for reſer, read refer.
6, 12, for i, read is.
8, 7, for exiſtance, read exiſtence.
9, 7, for ſubſtances, read ſubſtance.
23, 4, for contents, read content.

Plate I.

Fig. 1.

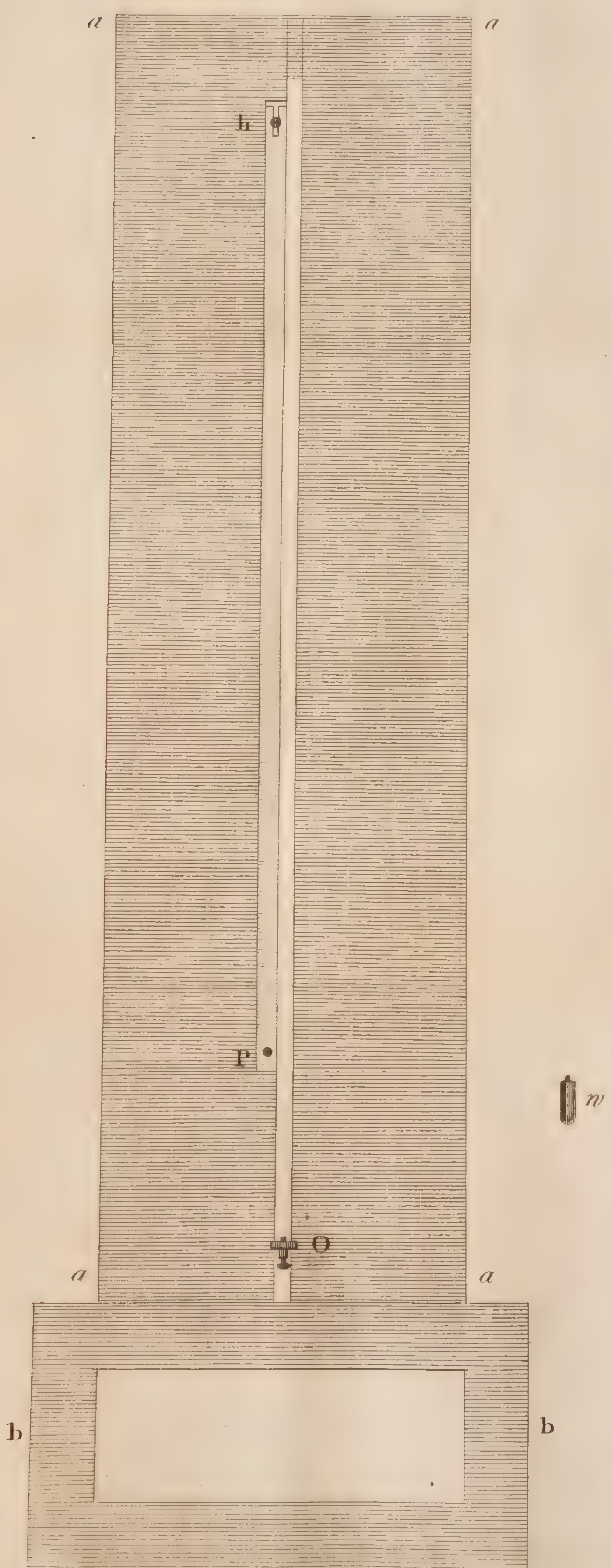
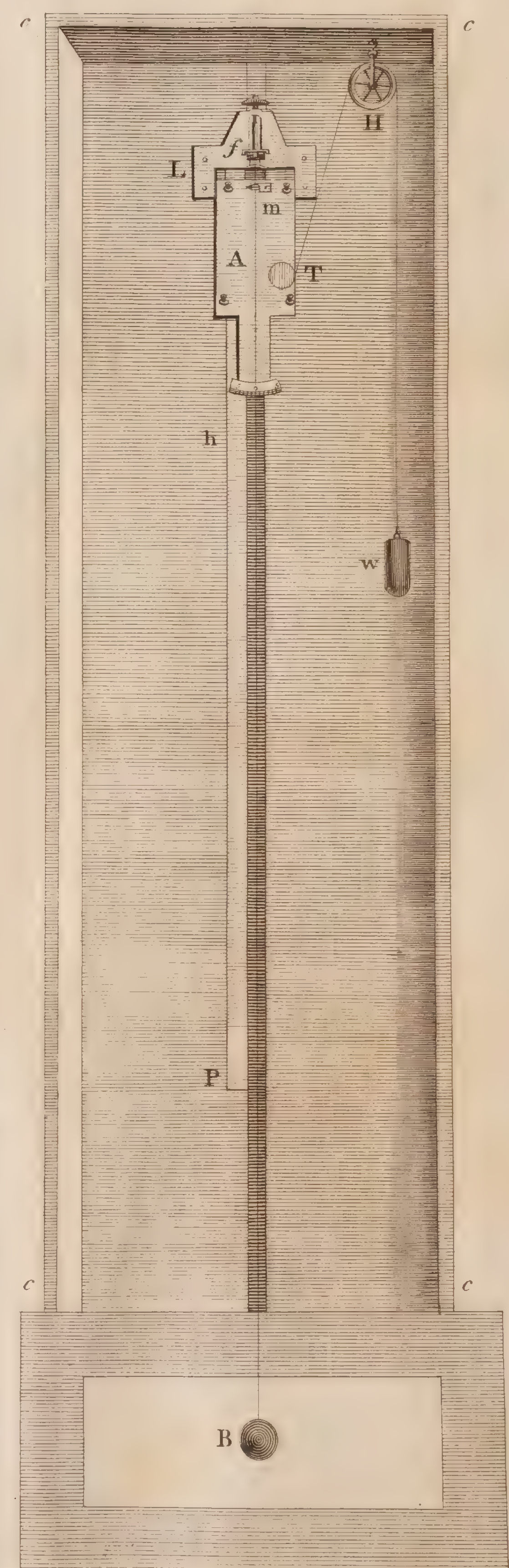
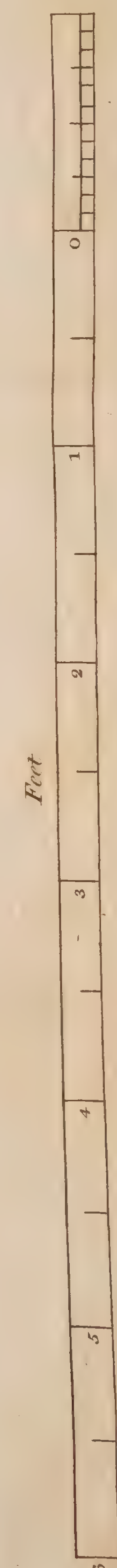


Fig. 2.





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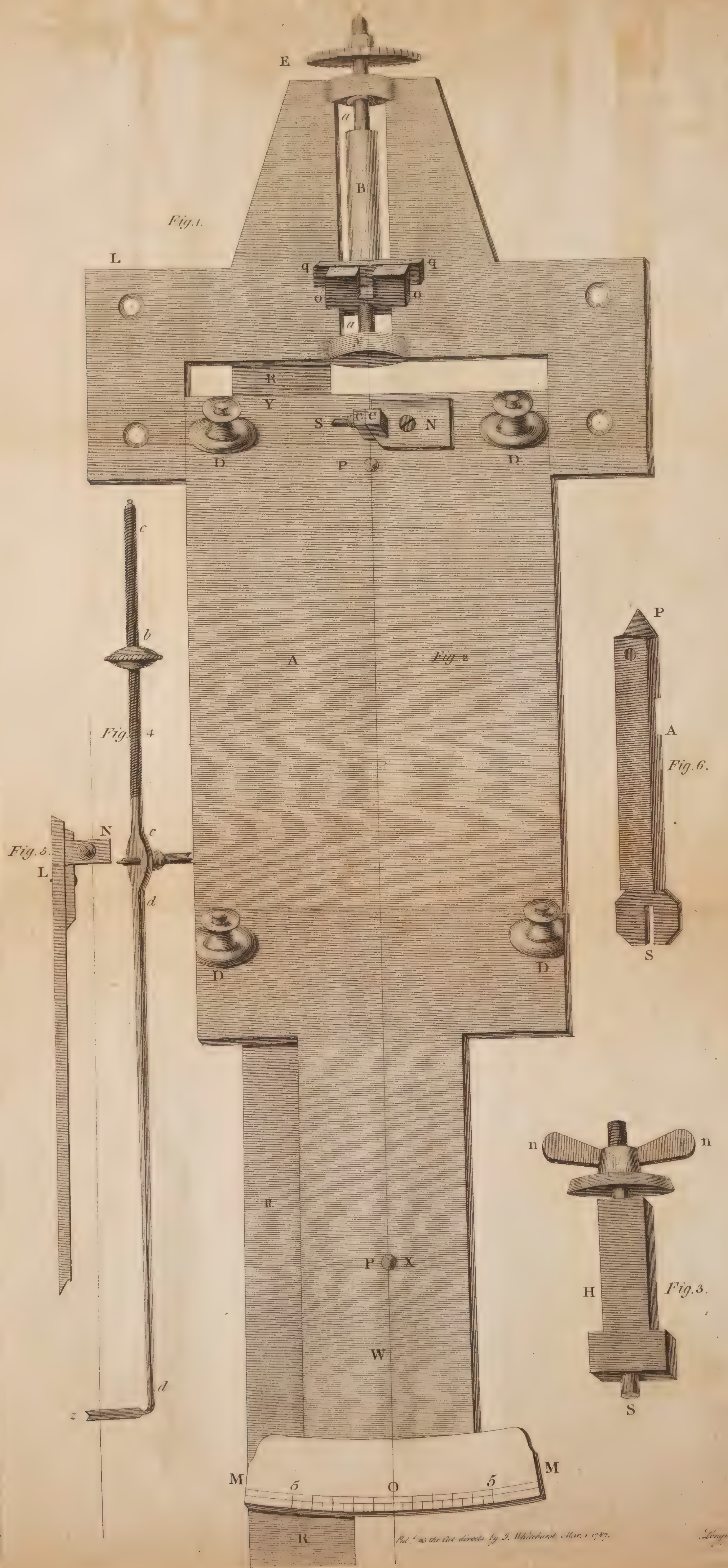
Longmate sculp.



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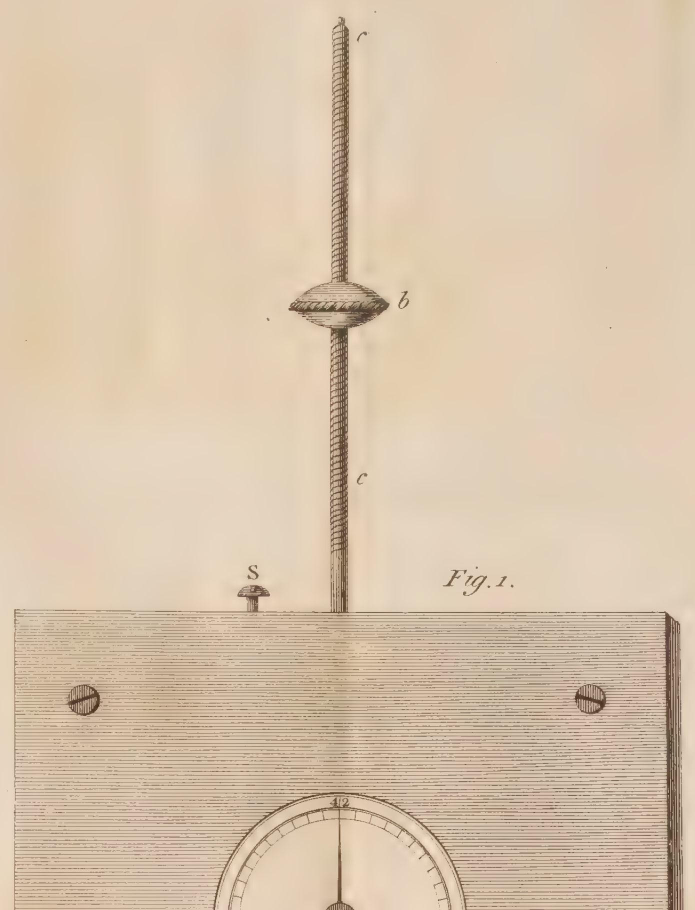
Longmate sculp.

Plate III.

Fig. 1.



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Longmate sculp.

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